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FORMS AND METHODS OF DIGITAL MEDIA APPLICATIONS IN MATHEMATICAL EDUCATION

Abstract

This article undertakes a brief review of the traditional, but in many cases modern forms and methods of working with students on mathematics lessons. The emphasis has been put to digital forms and methods of supporting mathematics teaching: e- learning, m- learning, WebQuest and the use of online resources of mathematics websites WWW. An attempt has been made to demonstrate the impact of digital media on changing the style of teaching mathematics. Only relevant and effective forms and methods of the use of digital media can bring the effect required. Mathematics will become more user-friendly, accessible and understandable; will enable young people to overcome fears caused by the lack of mathematical knowledge.

Key words: computer, digital media, education, forms of teaching, functional teaching, problematic teaching, mathematics, mathematical information technologies, informatics, methods of teaching, excel, GeoGebra, e-learning, m-learning, WebQuest.

Introduction

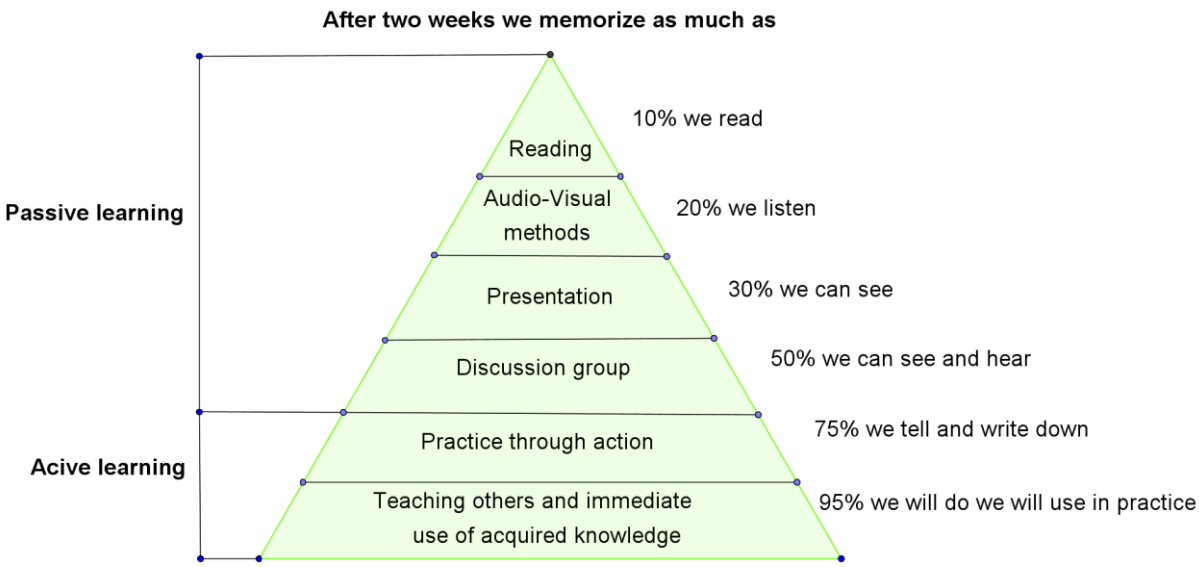
The use of different forms and methods of working with students results in new teaching and educational situations which support the development of individual personality. Let the concept of form and method of teaching serve as an introduction to discussions on forms and methods of teaching with the use of digital media.

The term form of teaching means the organizational side and includes external conditions of the process. We are talking here about the relations between students and teachers combined into appropriate groups, cooperation within the groups, and individuals with one

another, the types of activities as well as time and place of teaching. In the macro scale, the form of teaching refers to such organizational categories of students' work during the lesson (except lesson) as a collective (frontal) group or individual (unitary) work (W. Okoń, 1992, p. 59). The form is created by the constant learning content together with a combination of work organization and a method "The teaching method it is the way systematically used with students to enable the achievement of learning objectives, […] proven system implemented deliberately by activities for teachers and students to cause the assumed changes in the student's personality" (W. Okoń, 1992, p. 121).

Methods and forms of teaching in mathematics lessons

Before starting to work on the methods in mathematics lessons, it is worth looking into the results of studies on the way of receiving the knowledge and the amount (expressed in %) of knowledge that has been taught. The reception is often shown by means of so called pyramid of learning (dale's cone).



Drawing 1. The pyramid of learning
Source: prepared by the author.

It is assumed that on average 20% of knowledge is perceived by ear; up to 30% by sight, thus hearing and vision provide 50% to absorb the transferred material. If we add action to this, we can control up to 90% of knowledge. Therefore, it is necessary to pay attention to the important role played by activity during the process of learning. This division is not identical for all. People are divided into three groups:

- visual – people with visual representation, strongly responsive to visual stimuli;
- verbal – people with verbal representation, better react to auditory stimuli;
- kinesthetic – people with movement and touch system of representation, feel the best in action.

The following table shows a sample combination of features which are supportive and interfering in learning for different systems of representation.

Table 1. Juxtaposition of assisting and disturbing elements in learning for people with different systems of representation

	Help in learning	**Disturb in learning**
Visual	means of visual communication observation of presentation using graphs, schemes, pictures, making lists, notes, drawings	disorder, movement interrupting the presentation with words long explanations
Auditory	loud repetitions learns easily through discussions and lectures better understands what he/she reads if he/she can read the written text in a low voice solving problems through "loud thinking' or talking with others	noise too long and detailed written texts, a need for telling the stories about what can be seen or felt being forced to eye contact
Kinesthetic	specific actions, handiwork playing and presenting the material in practical way the ability to move during the listening, trips using models, objects that you can touch, feel and see how it relates to the description, the possibility of self-experimenting	the necessity of listening to too long explanations talking about what he/she can see or hear some verbal comments during the presentation being forced to remain motionless for a long time

Source: *I have a family*, http://mamrodzine.pl/strefa-rodzicow/wiedza-i-inspiracje/quizy-poznawcze/110-jestes-wzrokowcem-sluchowcem-czy-kinestetykiem [29.01.2012].

The awareness of different types of intelligence is important from the teacher's point of view. H. Gardner[1] lists the following types of intelligence: linguistic (linguistic skills), mathematical and logical (abstract and logical thinking), visual and spatial (visual forms of communication), musical (sense of rhythm), interpersonal (communication skills), intrapersonal (high self-esteem), kinesthetic (manual skills).

When you decide on using a teaching method you should take into account the perceptual and psychological abilities of students. Various teaching methods should be applied. Such approach will allow you and your students to reach educational success. Each method consists of a preparation phase and working with students. The emergence of new teaching methods

[1] H. Gardner (born 1943 in Scranton, Pennsylvania, USA), an American psychologist, a specialist in cognitive psychology and the psychology of learning, a lecturer at Harvard. He is known for his theory o multiple intelligence.

does not generate homogenous forms and educational methods. In the literature of the subject various divisions can be found. According to the sources by Kupisiewicz there are three groups:

- verbal methods – students gain knowledge by means of spoken or written word;
- bservation method – the source of knowledge is the surrounding reality or the teaching aids which can replace it;
- practical methods – the knowledge is gained owing to own practical activities associated with mental operations (Cz. Kupisiewicz, 2006, p. 31);

The concept of teaching-learning according to W.Okoń classifies methods as follows:

- giving methods (assimilation of knowledge) – learning by assimilation: talking, discussion, lecture, working with a book;
- problematic methods (independent investigation into knowledge) learning through discovery, classic, problematic method, method of cases, situational method, exchange of ideas, micro- learning, educational games;
- valorization methods- learning through experiencing ; impressionistic methods, expressive methods;
- practical methods- learning through activities; instructional methods, methods of task implementation (W. Okoń, 1992, p. 121).

The latest pedagogical research studies point to the following methods:

- giving;
- problematic;
- exposing;
- programming (software);
- practical.

A lot of attention has been paid to functional method in teaching mathematics (H. Siwek, 2005, p. 45), making use out of searching methods i.e. those in which the active part in the classroom should be primarily a student whereas a teacher should play a role of advisor and instigator. The main goal of this method is the acquisition of knowledge by the student not by means of solving more or less schematic tasks but by the well-planned actions organized by the teacher. According to the methodology of teaching, mathematical analysis of functional operations, inherent in the concept, should be accomplished, and then all kinds of exercises ought to be planned. Such exercises will enable the student’s transition from concrete

operations through imaginative to abstract ideas. In other words – analysis, understanding, practicing and the transition from the concrete to the abstract.

An extension of the functional method is problem-functional teaching. The problematic learning is used when difficulties will appear, which cannot be solved using popular and available schemes, rules and algorithms. In a sense, problem-based teaching makes the process of teaching closer to scientific research. It should be noted that this is the same as the teaching strategy or student's activity. All descriptions and assumptions emphasize the importance of ensuring the student's versatility and variety of acquired knowledge and skills as well as it should be remembered that the student ought to be the central figure of 'our art' and we teachers – directors and advisers.

When we plan a teaching methodology, one more problem should be taken up. The transmission of the information to students and then the creation knowledge out of it. The possibility of learning certain skills, in not limited to substantial issues, directly related to the contents involved in the subject which is taught. The development of the so called key skills in students is equally important (out of content), so these which will allow the individual to take full advantage of the knowledge as well as to rediscover yourself and move freely in the world around us, in the information society. The list of key skills is very long, about thirty are distinguished. Here are some of them:

- ability to work in a team;
- ability to use information technology;
- ability to communicate;
- ability to solve problems;
- ability to listen to others and taking into account their point of view;
- ability to use different sources of information;
- communication in several languages;
- linking and organizing various parts of knowledge;
- ability to organize and evaluate their own work.

If the teacher wants to achieve the objectives, he/she must find the most secure and proven ways and methods, which lead to the finishing line. It seems that the most important issues related to the general methodology of teaching mathematics can be combined into following groups:

1. Which methods should be applied (what does it depend from?).
2. The use of variety of resources and teaching aids.

3. Individual or group work.
4. Methods of activating students:
   - dialogue of a teacher with the class;
   - activities in groups (peer group participants, a group ‘with a leader’);
   - educational games;
   - student’s projects.
5. The introduction of mathematical methods to the areas close to the student (including age group and interests of the students) and real examples describing reality of the mathematical tasks.
6. The use of digital media as:
   - means facilitating the calculations (the role of programmer, formulation of algorithms);
   - visualization methods – graphs of functions, geometry in the plane and space;
   - sources of information;
   - means of individual exercises;
   - testing tools;
   - tools to construct mathematical models, dynamic structures.

While reflecting on the methods of teaching mathematics you should be aware of some basic facts and really reenact the already well known. When we start working with students in older classes, we can face a negative attitude of some students to the subject of the course. It is usually the effect of mistakes in the earlier stages of education and simultaneously the result of students’ submission to the opinion that ‘mathematics is difficult’. This opinion, imposed on mathematics at the very beginning of learning it, has a very negative impact on the further teaching of the subject. If a teacher of primary education has doubts as to his/her own mathematical talents, he/she will be more likely deal with other activities. Thus, such a teacher will spend less time on mathematics. In the vast majority of cases children have a great understanding of various mathematical problems and can draw appropriate conclusions. They should only be given a chance i.e. the correct method, form and knowledgeable advisor/ teacher.

Next problem that should be noted is confusion of the concept of ‘misunderstanding’ with the lack of basic knowledge. Certain facts and declarative knowledge are necessary to be able to move to the operational or procedural actions. Students should be made aware that without memory control (memorizing) of the basic information, it is impossible to proceed to the next

stage of learning. At this point the Internet resources can be supportive; however, the student must be aware of searching aims and dependencies (collectivism). At this stage, as at all the stages, the strengthening of the acquired information/ knowledge through exercises is very helpful. The next step is to check whether the student understands the transferred and accumulated knowledge. The approach is very important. If a teacher is sure that the student has a declarative knowledge but says that he/she does not understand, the teacher should give the student an easy problem to solve that requires the use of that knowledge. Then, very often it turns out that the understanding is not so bad. Thus, you can go to the more difficult issues. The majority of tasks should include issues related to the real-world situations, taken from everyday life. Such real examples make the lesson mare interesting for students and also provide answers to the frequently asked (in case of mathematics) question – "What do we learn for?", "What do we need it for?"...

A constructivist model of learning prefers independent investigation to solve a problem. A teacher, solving tasks on the blackboard and students 'obediently' coping such actions, would not make students think independently. Even if the teacher is the main person, who conducts the process of reasoning, it does not mean that he/she is relieved from activating the students from creative activities. In principle each teacher's statement should take a form of a dialogue with participants of the course and not only the selected ones but with their wide representation (a common platform of working).

In many situations, especially when a lesson deals with revised tasks (sometimes new ones); a group work gives very good results. This requires careful preparation of the tasks, rethinking of how to divide, handle and control the material. Planning the group lesson, we have to observe the regimes of time so that we have enough time to summarize the activities of various groups and their comparison. Educational games, mathematical crosswords puzzles or dominoes are ideal for group activities. You can use a variety of teaching aids – boards (charts), i.e. all elements of visualizing. Parts of the drawing or reasoning stages have to appear gradually. Students 'can see' better if the drawing is created in front of their eyes and they have the mathematical model.

Students' projects are interesting and popular method of teaching. They are carried out by a student or a group of students under the supervision of a teacher. The form of the project, its duration and the presentation of the expectations of the final results, first of all depends not only on the age of the students, we work with, but also on their interests. It is a method which enables to realize not only the substantive goals but also many key objectives. Moreover – what is very important – when applied properly, this method allows the weaker students, who

often using other method reach poor results, to demonstrate their knowledge. Applying this method we can create teaching aids, which will support teaching of mathematics. The brainstorming and a number of other activating methods such as: mental map, snowball technique, decision tree, meta plan, rug of ideas or six thinking hats allow students to raise problems by themselves, seeking solutions, hypothesis or skills for scientific discussions.

At this point a computer or an interactive whiteboard should be introduced (digital media) as tools to support visualization of mathematical images (spatial geometry), processes and stimulations (analysis of function) (J. Kandzia, 2011, p. 165-197). Using such a function perfectly fits all the methods and forms of teaching applied in mathematical education. It should be remembered that regardless of the form, method or means, experience plays a great part in gathering information and then converting it into knowledge or developing skills.

Digital forms and methods supporting the teaching of mathematics

E-learning. The education is changing together with the development of technology. We are witnessing the appearance of new method of teaching: e-learning. In Poland this method has been developing very slowly. The main reason is the poor infrastructure of information technology in our country. However, observing the impetus with which the e-learning has been distributed in the West, the success of this method of teaching in Poland seems to be only a matter of time.

On 20 January 2012 the Ministry of Education sent a draft of resolution for consultation to the Council of Ministers of the Government concerning the development program for students and teachers' competence in the application of ICT communication technologies. This program involved 'Digital school' together with management project of the Council of Ministers on the conditions, forms and implementation procedures concerning the development both students and teachers competences in the use of ICT. The implementation of this task aims at covering the period 2013 – 2016. The objective of this project is to achieve the functionality of the digital school in developing students and teachers competences with the use of ICT in education as well as acquiring the necessary competences in order to function in the society of knowledge. The project covers four areas:

- e-teacher – preparing teachers to teach, communicate with students and parents and keeping school records using ICT;
- e-educational resources, including e- book – supplement of the offer for public electronic educational resources including the access to free e-books;

- e-school – providing schools with the necessary infrastructure, in particular modernteaching aids;
- e-student – providing students, in particular those at risk of digital exclusion, with access to modern teaching aids (*Digital school*, 2012).

History of distance learning has more than a century old tradition. Such names as J. Steward and I. Pitman are listed as precursors of correspondence education. In 1883 the university of distance learning was established and in 1890 the International Correspondence School. Along with the popularization of radio at the University of Iowa in 1925 new educational programs were established. In 1940 the University initiated educational television. The first educational programs have been created with cooperation of the Stanford University with IBM.

Australia is a pioneer in non- standard teaching. In the early 1950s the so called School of the Air was launched. The development of technology causes replacement of the radio by other media – satellite communications, broad band Internet connection, HDTV television, video technology, computers, systems and information networks, digital technologies, wireless telephony, multimedia, bio-computers. The devices except for sending information allow to cerate, store, select and playback almost immediately. New perspectives for virtual education are opening; the emergence of a global technical university is being talked about (J. Kandzia, 2013, p. 279).

Distance learning through the Internet constitutes a very attractive alternative for the traditional teaching. It enables learning at the very high level which can be available at any place and any time. It gives a chance to people with disabilities, working people and all those looking for knowledge 'accessible everywhere and all the time'. In the corporate and public administration such learning reduces costs and gives access to 'the real knowledge at the right place and the right time' (so called just-in-time learning). The complex information systems are no longer able to determine the level of knowledge of each trainee and as a result to adapt the process of learning to individual needs (so called learning the differences).

In Poland e-learning appears under different names: e-courses, e-learning, remote control. Thus, it fits into the idea of lifelong learning very well (*Lifelong Learning…*, 2013)[2]. The terms and models related to e-learning include such as:

[2] Lifelong learning (Brit. Lifelong Learning Program- LLP) – educational program put into operation by the European Commission in 2007 as a continuation of the Socrates Program II. LLP came into being in the years 2007-20013. It aims at strengthening cooperation between the European Union and supporting the exchange of students and teachers within the member countries. The National Agency of LLP program in Poland is the Found for Development of Educational System.

1. Online learning – learning with the use of the Internet (lack both the physically existing teaching materials and the direct contact with the teacher).
2. Mixed-mode/resource-based/blended learning – a method linking direct teaching (through face to face contact with the teacher) with distant learning.
3. Learning object – a program, page, file that is used many times during the e-learning.
4. LMS (Learning System Management) – a set of e-learning tools available through the common interface. This system often takes the form of educational platforms. Database – searching for information in the indexed databases.

Online Support – similar to the database, however, the source of information embraces forums, newsletters and electronic mail. Asynchronous training – involves learning at the learner's own pace; contact with the teacher through discussion groups, e-mails and forums; all materials are available in the electronic form. Synchronous training- listeners communicate with the instructor at a certain time; in the form of virtual class or a video conference.

M-learning (Mobile-learning). An innovative, more modern form of e-learning using wireless technologies: laptops, smart phones, tablets, mp3.This is a wide range of portable computers (laptops, notebooks, subnotebooks), connecting features of mobile devices and desktops – the same operational systems, dimensions, sizes and screen resolutions similar to the traditional ones. The advantage of these devices: unlimited mobility (small sizes that allow keeping in hand and carrying in a pocket), small energy demand (allowing for many hours of use without access to the electricity gird), miniature screen (with sizes much smaller than the screen of traditional computer or television) (L. Hojnacki, 2006, p. 23, 27). You can have them always with you. It is predicted that their impact on the society can be much bigger than the traditional computerization. They are the driving force in our information society.

WebQuest is a method involving the use of online web resources WWW. Students/listeners search for some information on a given topic, digest and build on them the basis of their knowledge. It is a form of group activities but this method can also be used in distant learning and individual research in the field of informal education system and also in the lifelong learning. It is more effective in teaching than the traditional methods. Learners can play different roles, contact through e-mails with eminent personalities of the world of science, art and politics or through organized teleconferences can give the opportunity to participate in staged situations according to the proposed scenarios or to create their own multimedia presentations on the Internet pages. This method stimulates the brain and activates various mental skills: comparing, classifying, inducing, deduction, analyzing errors, constructing arguments, abstracting, analyzing views.

The author confirmed the advantages of this method, which makes use of the Internet information. She has conducted natural pedagogical experiment – mathematics lessons with the use of mathematical resources on the network. She put a question what knowledge and skills the students acquired in the course of learning with the use of computer supported techniques and more specifically by using information from the Internet. Does the use of the Internet during mathematics lessons help to shape the students creative attitude towards the world? Which cognitive activities are formed by the Internet? (J. Kandzia, 2011)[3].

The aim of the study was to demonstrate relationships between the introduction of the Internet into the learning process and the creative activity of the youth, which are the most desirable correlations in the activity of a modern man. The results of the research indicate that the use of this new tool forced students to complex mental activities, self observation of certain conditions, which has been imposed by reality, changes in the general laws and the necessity of anticipation of new solutions under the changed circumstances. This form of education helps to build creative attitudes of students as well as to create conditions in order to reveal such attitudes. The youth learnt to perceive diversity of the world and how to break the stereotypes of its perception. They were taught to accept the 'new' and to notice the importance of their own actions. This in turn triggered exploratory activity forcing them to improve the status quo. These lessons helped many students to overcome the fear of mathematics or even aversive attitude to the subject. They developed divergent, intuitive and multi-faced thinking (J. Kandzia, 2011, p. 102).

During the experiment students have learnt to trigger cognitive activity, thus to select, analyze and present, in a synthetic form, the knowledge gained in the process of education. They have learnt to discover some information and connect them into a new meaningful entity. They have been forced to seek common explanations of various facts and check their usefulness in practical activities – solving problems. They have determined correlations between the symbols and relevant phenomena in the real world (J. Kandzia, 2011, p. 105, 106).

A question has also been put: Which didactic and educational values can be achieved using this medium? There are some of them. Using the Internet (individual searching) in the mathematics lessons teaches self-approach to discover and define new truths about the world (noticing and solving problems), teaches to determine correlations between symbols and correlating phenomena in the real world, teaches also to construct actively their own

[3] The experiment and its results have been described in the author J. Kandzia's book: Forming didactic and educational values in teaching mathematics with the use of multimedia techniques.

knowledge and how to reorganize their own cognitive mental structures, teaches to define correctly and clearly important notions and functional dependencies, creates cultural behavior and socially acceptable behavioral pattern, affects forms of mutual acceptance and understanding between a teacher and a student, introduces order and regularity of student's work (J. Kandzia, 2011, p. 118). To build the project of a lesson according to WQ method you should:

- to formulate a subject of the lesson;
- to browse the Internet pages in terms of data availability;
- to organize, categorize and group the knowledge and materials obtained from the Internet in the form of: 1) pages www and data base (online resources), 2) useful materials and projects resources, 3) human resources (experts online and local experts) bee;
- to prepare a conspectus (J. Kandzia, 2011, p. 158-171)[4].

The impact of digital media on changing the style of teaching and learning

The use of technological devices as an aid in teaching mathematics is a fact. However, it should be remembered that all previous didactics in mathematics, all styles and methods of working with students were constructed on the traditional basis, well – known and frequently checked patterns. The learning objectives both general and directly prospective have been taught using certain patterns of work that are effective and efficient.

In case of new technologies that support the mathematics teaching, the tool is changing. At the same time the way of teaching and learning has been accepted. However, the aims achieved are supposed to be the same as before. This is not always possible. Teachers often rely on the old, traditional patterns to which they "add" new tools such as multimedia.

The predecessors of today's computers were abacuses (soroban used till now in Japan), calculators (built by Pascal and Leibniz), and mechanical engines (Babbage's difference and analytical engines). They served mainly to perform operations on numbers. The primary purpose of digital and mathematical machines was to deliver mathematical calculations. The development of computers contributed to the emergence of a new field of knowledge – computer science. The fastest growing part, which deals with the basics of computer calculations, is algorithmic. We are talking here about a sort of equilibrium which exists between the information and communication technology. Information as an object is subject

[4] A fragment of the article by J. Kandzia, 2013, p. 118-122.

to action and communication, which are the destination of this information and the main aim of the action.

Calculating sciences are a new interdisciplinary field of knowledge. They deal with the use of computers in the analysis of scientific problems from different areas. Supercomputers are created, the power of which must be supported by computational method. Mathematics and computer science deliver mathematical models as well as effective methods for solving problems of the modern world (M.M. Sysło, *Komputery, Informatyka…*, 2014; fragment of article by J. Kandzia 2011, p. 94).

Information technology (IT) is a response to the constantly expanding use of computer science in our society and the increasing role of computers in communication and information exchange. Information technology (IT) is:

- a set of means computers, their peripherals and computer networks;
- tools – software;
- other technologies i.e. telecommunication which are used for handling all the information.

Therefore, includes within its scope – information, computers, computer science and communication. The modern technology has grown on the basis of computer usage. Its decisive role for life of societies allows defining the end of the twentieth century as the era of information and its technology (E. Gurbiel., G. Hard-Olejniczak, E. Kołczyk, H. Krupicka, M.M. Sysło, 1999).

Still growing primes which are being obtained by supercomputers can serve as examples of the impact of information technology and its enormous causative power for mathematical achievements. The Internet is an example of cooperation of thousands of computers on generating further Mersenne primes (E. Grubiel, G. Hard-Olejniczak, E. Kołczyk, H. Krupicka, M.M. Sysło, 1999; D. Harel, 1992; M.M. Sysło, 1997; M.M. Sysło, 1998; J. Madey, M.M. Sysło, 2014). The last record primes (the largest number obtained is: 2, 57885161 – 1)[5] have been generated on IBM PC computers. The equipment that supports calculations is on one hand based on the properties of operations and in general – algorithms, for which they have been designed and on the other hand, they have had and still have a big impact on the development of calculation methods and mathematics. The second part of this statement refers in particular to computer software. Computers, computer calculations and

[5] 8th February 2013, it contains more than 17 million digits, 48 Mersene's number. It has been generated by combining 360 processes http://technologie.gazeta.pl/internet/1,104530,13366731, it is the latest prime over 17 millions.html.

computer mathematical methods have become an integral part of mathematical sciences. This can also refer to calculators, albeit to a limited extend.

Computers in teaching mathematics are part of the mathematical knowledge and equipment that the student is (should be) prepared to use at learning mathematics on everyday basis, both in the process of acquiring education and lifelong learning or in the future work. The use of mathematics and information technology, the combination of mathematics with computers are an integral part of interdisciplinary teaching. They form the basis of teaching methods using digital media. The ability of using computers constitutes an example of good practice in their application (E. Gurbiell, G. Hard- Olejniczak, E. Kołczyk, H. Krupicka, M.M Sysło, 1999). The use of digital media in mathematical education provides new additional opportunities to communicate; they support creative development of the learning person. Their function involves primarily a teaching aid and subsequently a component of mathematics i.e. such a device, which in combination with methods of informatics, extends its scope and methods.

M.M. Sysło distinguishes four groups of computer links between the contents and forms of communication that are taught: support (the use of program to analyze the graph of function) and enrichment (visualization of spatial geometry, simulation of changes for various calculations) traditional forms and contents of communication; new possibilities (data collecting and graphical representation of dynamic geometric constructions) and contents within the scope of traditional teaching and skills (the use of spreadsheet for statistical calculations, constructing mathematical models and different phenomena as well as their digital stimulation) (M.M. Sysło, 2014).

Examples of the use of digital media in mathematics lessons

The dynamic mathematical program GeoGebra, which the author uses with students, both in high schools and at multimedia workshops, can be an excellent example for the visualization of problems. This software connects algebraic, geometric and statistical options. It can be used to visualize mathematical concepts and mathematical correlations as well as to deal with numerical calculations and algebraic simplifications. Dynamic structures consisting of points, vectors, sections, straight lines, conic graphs, and graphs of functions can be created in a simple way. It can find spaces for geometric points perform geometric transformations, statistical tasks, use them as a spreadsheet. GeoGebra allows animating objects, creating your own tools based on the existing structure. Markus Hohenwarter, together with an international team, deals with this software. This program is constantly updated and can be expanded by

everyone and is free for teachers, pupils, students and people interested in mathematics (J. Kandzia, 2011, p. 182-189). The use of spreadsheet in mathematics lessons not only allows to check correctness of solutions obtained by the traditional methods, but also to broaden mathematical knowledge aiming at using computer techniques. The requirements for both teachers and students come down to a basic knowledge of Excel. The task is feasible within one-hour lesson (J. Kandzia, 2011, p.176-181).

The author wrote previously about the use of the resources www pages, thematically related to mathematics (J. Kandzia, 2012, p.166-175)[6] An interesting alternative to the traditional textbooks are mathematical computer programs online and also those, which can be installed on your computer as well as teaching materials on CD provided by the publishers. They are great tools that enable understanding and fixing the problem of a certain task. Simulations of various problems allow students to observe the correlations that exist among mathematical objects. Visualization on the computer screen brilliantly brings closer problems associated of with spatial vision which causes big problems. They diversify the process of learning and teaching. It is possible to calculate easily, to create teaching aids (and not only), to draw and transform. Students can find errors themselves. However, the program without knowledge and teacher's activity is useless. The students can be convinced that learning mathematics can be as interesting as their favorite computer games. (J. Kandzia, 2012, p.198-200).

Thus, at every stage of mathematical education, computer studies and information technology can support both the existing model, erect new opportunities within technological contents and deliver new ones as well. Preparing for life in the information society requires knowledge and skills in using the tools in the future. The computer, which is an element of information technology, constitutes a means of communication and a source of information for both the teacher and the students. Testing student's achievements, designing tests or using online educational services cease to be enough for the traditional education. It becomes (is) a teaching aid. The educational success can be achieved under condition that there is a full cross – curricular integration i.e. integration in the use of information technology in every sphere of teaching and learning related to mathematics and the ability to work with technology. Such standard teaching practices like: putting computers 'next to' a student, taking into account the use of textbooks, integrating curricula, teaching aids (textbooks, computers and software) and methods of their usage in teaching mathematics are insufficient.

[6] Examples of lessons with the use of didactic materials.

The core school curriculum, among the general tasks, includes the following statement: "Teachers create conditions suitable for students to acquire the following skills […] searching, organizing and using information from various sources and also for the effective use of information technology" (*Bip MEN*, 2014).

Summary

To conclude the reflections on forms and methods of teaching mathematics, using both traditional and modern methods, some common links and features cannot remain unnoticed. In the digital age the most welcome methods are valorization methods and the methods which require problematic approach including the project methods as well as activating methods. Provoking students to activities; constructive, creative, problem solving, searching skills and selecting information, analysis of information, drawing conclusions, exploring, defining. In a word, building knowledge based on the acquired information. This allows both the teacher and the students, to achieve full satisfaction and fulfillment in the certain field of science. Digital media perfectly fit into the current methods of teaching mathematics. "We can see", gain some information, process them, we do not waste valuable time on laborious calculations; perform simulations at different values of data. Teaching and learning take place in an unconventional way; the student is the main link in the educational process. E-learning forces both individual (individual pace of work) and group form of work. The philosophy behind this type of learning is the ability to cooperate, compete in the group, tolerance, balanced judgments, and lack of prejudice, respect for different views, regularity, and ability to ask questions.

Defining the role of media in the learning process and identifying the extend of their impact on the way of teaching and learning processes, it is worth recalling McLuhan's views: "the shaping force of a means is this means itself" (M. McLuhan, 2001, p. 54). He points to a number of specific characteristics of 'electric' techniques which "refute space and time; they are instant and ubiquitous, they produce numerous centers without margins; indicate the need for structural approach to all the knowledge; favor not what is fragmented but what is organic; do not create a nation but a tribe- not a superficial correlation of equal units but the coherent model of fully related kinship groups which are deeply decentralized and distributive when it comes to their psychological and social effects" (M. McLuhan, 2001, p. 228). In these statements many aspects of our reality can be perceived. The conservative attitude to technological changes ought to be rejected but forms and methods of teaching should be treated reflexively so that they were consistent with the requirements of "the age of

electricity". Such approach applies both to the didactic policymakers and also to all the subjects related to education. Hence accurate and effective forms as well as methods of digital media applications in teaching and learning mathematics.

The Internet access is a very important factor in developing students' knowledge, increasing the overall effectiveness of teaching which also shapes the active approach to learning process so much needed in their working life (R. Tadeusiewicz, 2000). Digital media are not a magic cure for solving all the ailments and problems of education. They are only a very efficient tool with the help of which educational contents can be delivered easier and more effectively, a study path can be individualized to make the educational process alone more interesting which stimulates students to creative actions, generates their skills and habits needed in adult social life. On the other hand, the contents transmitted and controlled by this tool have to remain in teachers' hands who will ultimately determine the purpose and direction of education (J. Kandzia, 2012, p. 58).

Speaking about the methods and forms of teaching the self explanatory words of Confucius should be reconsidered: "Say and I will forget. Show and I will keep in mind. Let me participate and I will understand".

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